A new kind of science

This is a book about mathematics. It is easy to read. It is not a popular survey of old knowledge but an exposition of a distinctive idea by a leading thinker. What it says is true (give or take some of the more speculative parts). That is a combination that does not come along every day. It is quite enough to make the book well worth reading, however irritating some of its features are.

The distinctive idea is easy to state, though it needs some examples to appreciate. It is that simple rules can generate complex - very complex - outcomes when they are repeatedly applied. Isolated examples of this phenomenon have been known for centuries. For example, the number p has a simple definition - the ratio of a circle's circumference to its diameter - and there is a quite simple formula to calculate its digits. The result begins:
3.141592653589793238462643383279502884197169399375105820974944592307816406286208998628034825342117068…

There is no visible pattern in these digits. That initial intuitive impression can be confirmed by statistical tests that show there are about the same number of each digit, in the long run, that there are about the same number of runs up (like 159) and down (like 653), and so on. The sequence of digits exhibits the kind of complete patternlessness or randomness that can be generated by simple rules. Wolfram is more interested in complexity that is not quite so formless. He finds it especially in the mathematical objects called cellular automata. A classic example of these fascinating entities is "Langton's ant". Take a large checkerboard, with the squares initially all white. Start at one square and color it black, and move to the square to its east. Keep moving according to this simple rule: if the square you are moving into is white, color it black and turn left; if it's black, change it to white and turn right. The result is a gradually growing trail of black dots, of an intricate and unpredictable shape something like an ant trail. (There is an animation at users.libero.it/acnard/ant.html) The complexity of the shape comes not from any complexity in the cause, but from the complex way in which the trail intersects itself and heads off in new directions.

Stephen Hawking remarked that each equation in a book halves its sales. Readers will be pleased to learn that Wolfram does not believe in equations - not only because they impede communication but because he regards them as symptomatic of the old sort of mathematics that he wishes to move beyond. The weight of explanation rests instead on the text, and even more on the pictures. Wolfram has spent twenty years poring over computer-generated pictures that show the various kinds of complexity that can be generated by following simple rules repetitively. (A few samples can be seen at www.wolframscience.com/preview) He is a gifted scientific communicator, and an outstanding feature of the book is the selection of these pictures of complexity and their clear and simple explanations in the text.

The virtues of the book are most evident in Wolfram's discussion of the notion of randomness. It is a concept that has caused immense confusion, with a great deal of scratching of heads as to whether computer-generated random numbers or the digits of p are "truly random". As Wolfram explains, there are three quite different concepts of randomness: first, being generated by a stochastic or chance process, like the throwing of coins; second, being patternless; and third, being incompressible, that is, not being generated by any short computer program. What has not been much appreciated is that the second and third definitions, the purely mathematical ones, do not at all apply to the same things. The digits of p, for example, are patternless, but generated by a quite simple program. His distinction is sound, and clears up a lot of confusion. He also argues that the stochastic concept of randomness is something of a chimera - if we ask what is happening physically with coin throwing, will we not see it as like a computer program, the complex series of coin outcomes being another instance of complexity internally generated by a simple mechanism?

Where Wolfram is not so convincing is in his vastly ambitious project to demonstrate that his idea will revolutionize all of human knowledge, from fundamental physics to the philosophy of free will. In long chapters on evolution, physics, perception, developmental biology and so on he argues that science has hitherto taken a simplistic view of complexity, and that his own perspective both takes it seriously and explains without remainder how it arises. He suggests we see all the complexity in the universe as arising like that in the trails of Langton's ant - simply from the reiterated application of simple rules or programs. He argues that our ideas on complexity are biased by thinking about engineering, where "complex" systems are specifically designed by us to be simple enough so we can understand their workings. Nature does not operate under this restriction, he thinks, and so can let fly with really complex complexity. The difficulty is that the engineering type of complexity, where parts interconnect to some purpose, is the interesting kind. Galen in his ancient classic On the Usefulness of the Parts of the Body could discourse at length on the intricate connection of the parts of the hand and their usefulness for gripping, and conclude that such design pointed to a divine Designer. The same kind of modular, hierarchical structure, with parts working together to a purpose, is found in software, symphonies and societies. The promise of Darwinian natural selection - of whose powers Wolfram is rather sceptical - was that it could explain that sort of design as caused by an easily understood random search process. Wolfram is blind to that sort of complexity and there is no clue in his book as to how cellular automata could possibly generate it. He is more interested in intricate patterning of the sort found in the coats of tortoiseshell cats. It is useful for camouflage, perhaps, but is otherwise a pointless sort of complexity - an expression, shall we say, of the Jackson Pollock aspect of Nature's artistry rather than the Rembrandt.

Wolfram is right, though, to insist that natural scientists need to understand the possibilities of simple causes producing complex effects, so as not to conclude that there must be complex causes, merely because there are complex effects. Cellular automata must be part of the armory of large-scale science.

There is one area of applied science on which Wolfram's perspective does cast genuine light. Perception is what mathematicians call an inverse problem, or what philosophers call inference of causes from effects. The visual system has amazing powers to "drink from the firehose of data" - to see patterns in its mass of input, and infer from them the true properties of the objects causing them. It can infer 3D shape from a 2D projection, shape from shading, its own motion from optical flow. Seeing patterns in data is, Wolfram says, like inferring the simple rules of a cellular automaton from its output. It is easy if the output is homogeneous, or has simple repetitive structure like stripes or textures. It is possible with some difficulty for a few more complex patterns, like the nested or fractal patterns of ferns, but hopeless for truly complex patterns like those generated by the cellular automata he is principally interested in. That would be as hard as trying to guess the formula for p by looking at a stretch of its digits. He is again blind to the more "architectonic" patterns visible in, say, landscape paintings, which are visible to a trained perception that merges with aesthetic sensibility. Nevertheless, his approach to perception usefully sets the stage by placing the problems of perception in the correct abstract setting.

As many of the first readers of Wolfram's book have pointed out, it has many annoying features. He writes of the virtue of modesty "Perhaps I might avoid some criticism by a greater display of modesty, but the cost would be a drastic reduction in clarity." Just about anything in the mathematical theory of how systems evolve in time is called "my discoveries"; elaborate speculations about life, the universe and free will are preceded by "my strong suspicion is", as if that were a reason for believing them; one often wishes the text would stop repeating itself and exhibit more complex behavior. The scientific reaction is already shaping up the way it did to Thomas Kuhn's The Structure of Scientific Revolutions, of which scientists said, "His coverage of my area is certainly thin and ill-informed, but on all other parts of science he's most stimulating." The experts will complain, and rightly, but the general reader need not be too concerned. A book that generates a sense of excitement about new and comprehensible ideas in mathematics is an event worth celebrating.

A New Kind of Science is a best-selling, controversial book by Stephen Wolfram, published by his own company in 2002. It contains an empirical and systematic study of computational systems such as cellular automata. Wolfram calls these systems simple programs and argues that the scientific philosophy and methods appropriate for the study of simple programs are relevant to other fields of science.