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Spectral distributions of adjacency and Laplacian matrices of random graphs

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Abstract

In this paper, we investigate the spectral properties of the adjacency and the Laplacian matrices of random graphs. We prove that:

(i) the law of large numbers for the spectral norms and the largest eigenvalues of the adjacency and the Laplacian matrices;

(ii) under some further independent conditions, the normalized largest eigenvalues of the Laplacian matrices are dense in a compact interval almost surely;

(iii) the empirical distributions of the eigenvalues of the Laplacian matrices converge weakly to the free convolution of the standard Gaussian distribution and the Wigner's semi-circular law;

(iv) the empirical distributions of the eigenvalues of the adjacency matrices converge weakly to the Wigner's semi-circular law.

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